4 Sequences and Series

4.1 Finding Patterns

5/12/14

Complete yout Unit 4 pretest.

4.1 Finding Patterns

5/12/14

What are patterns? Where do we see them in everyday life?

Things that repeat, sequences

happens at the same time period

Seasons, holidays, birthdays, menstrual cycle

Days of week, months, hours

Street names

4.1 Finding Patterns

5/12/14

IWBAT identify the missing term in a pattern of pictures, letters, or numbers. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

4.1 Finding Patterns

5/12/14

1) S, M, T, W, T, F, S

2) J, F, M, A, M, J

3) A, E, I, M, Q
1 5 9 13

4) M, V, E, M, J, S

IWBAT identify the missing term in a pattern of pictures, letters, or numbers.

4.1 Finding Patterns

5/12/14

A) 1, 3, 5, 7

B) 4, 9, 16, 25

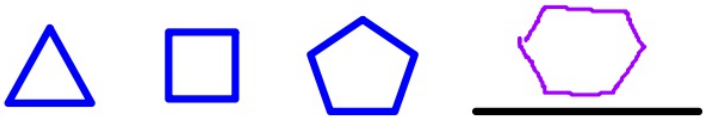
C) 1, 2, 3, 5, 7, 11

D) 1, 2, 3, 5, 8, 13

IWBAT identify the missing term in a pattern of pictures, letters, or numbers.

4.1 Finding Patterns

5/12/14

1) 

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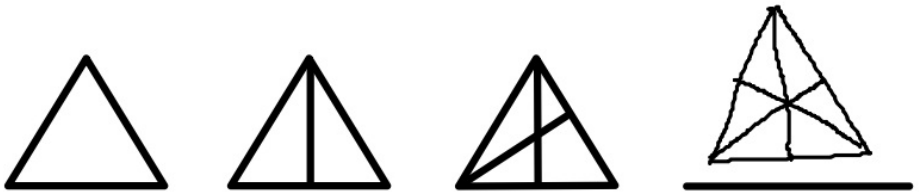
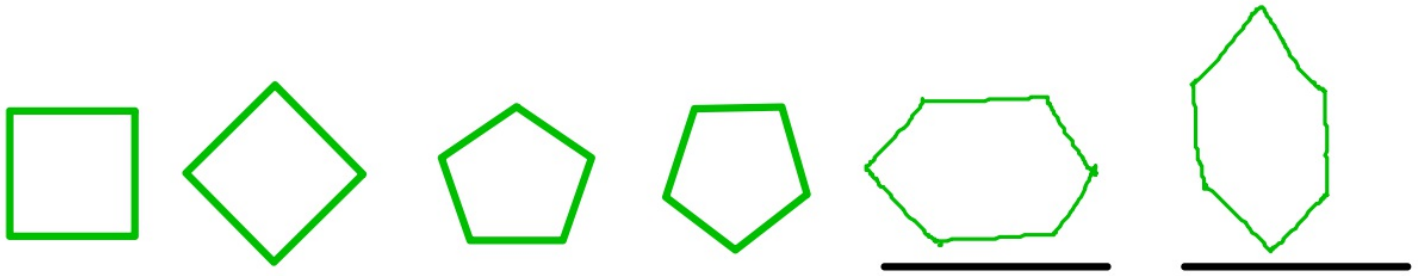
3) 

4) 

ant, dog, lion, ele, giraffe

IWBAT identify the missing term in a pattern of pictures, letters, or numbers.

Vocabulary 4.1.1 p. 16

Practice 4.1.2

Apex quizzes 4.1.3 & 4.1.4

IWBAT identify the missing term in a pattern of pictures, letters, or numbers.

4.2 Arithmetic Sequences

5/13/14

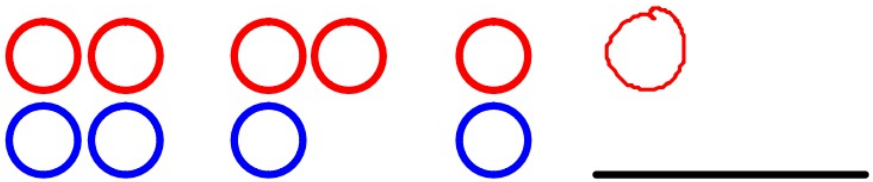
Identify the missing term in a pattern of pictures, letters, or numbers.

4^2 9^2 16^2 25^2

1) 16, 81, 256, 625, ...

2^{2^2} 3^{2^2} 4^{2^2} 5^{2^2}

2) A, C, F, J, ○

3) 

4.2 Arithmetic Sequences

5/13/14

Define and identify an arithmetic sequence.

An arithmetic sequence is a sequence of numbers that have the same thing added to them every time. This amount that is added is called the common difference.

Which of the following are arithmetic sequences?

1) 1, 2, 3, 4, ... common difference = 1

2) 4, 2, 0, -2, ... $d = -2$

3) 2, 4, 8, ... NO

4) 1, 4, 7, ... $d = 3$

How do you know?

IWBAT use the simple, explicit, or recursive formula to solve problems about arithmetic sequences; find the missing term in an arithmetic sequence; find the common difference between terms of an arithmetic sequence; and identify the graph that represents a given arithmetic sequence. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

Find the rule for the following sequence:

1, 4, 7, ... add 3

$n \quad a_1 \; a_2 \; a_3$

$a_n = a_{n-1} + 3$

What is the 4th term? What is the 50th term?

$a_4 = a_3 + 3$

$a_4 = 7 + 3 = 10$

$a_{50} = a_{49} + 3$

IWBAT use the simple, explicit, or recursive formula to solve problems about arithmetic sequences; find the missing term in an arithmetic sequence; find the common difference between terms of an arithmetic sequence; and identify the graph that represents a given arithmetic sequence.

4.2 Arithmetic Sequences

5/13/14

What term is missing from this sequence?

A) 1, 5, 9, 13 , 17, ... $d = 4$

B) 8, 4 , 0, ... $0 - 8 = -8 = -4$, $\frac{-8}{2} = -4$

C) 1, 18 , 35, ... $\frac{35-1}{2} = 17$

D) 1, 16 , 31 , 46, ... $\frac{46-1}{3} = 15$

IWBAT use the simple, explicit, or recursive formula to solve problems about arithmetic sequences; find the missing term in an arithmetic sequence; find the common difference between terms of an arithmetic sequence; and identify the graph that represents a given arithmetic sequence.

Given the explicit formula, derive the simple formula.

Explicit formula: $a_n = a_1 + (n-1) * d$

$a_n = a_1 + dn - d$

$a_n = dn + (a_1 - d)$

Simple formula: $a_n = d * n + (a_1 - d)$

IWBAT use the simple, explicit, or recursive formula to solve problems about arithmetic sequences; find the missing term in an arithmetic sequence; find the common difference between terms of an arithmetic sequence; and identify the graph that represents a given arithmetic sequence.

4.2 Arithmetic Sequences

5/13/14

Explicit formula: $a_n = a_1 + (n-1) * d$

Simple formula: $a_n = d * n + (a_1 - d)$

Recursive formula: $a_n = a_{n-1} + d$

Write the explicit formula, the simple formula, and the recursive formula for the following sequence: 4, 2, 0, -2, ... $a_1 = 4, d = -2$

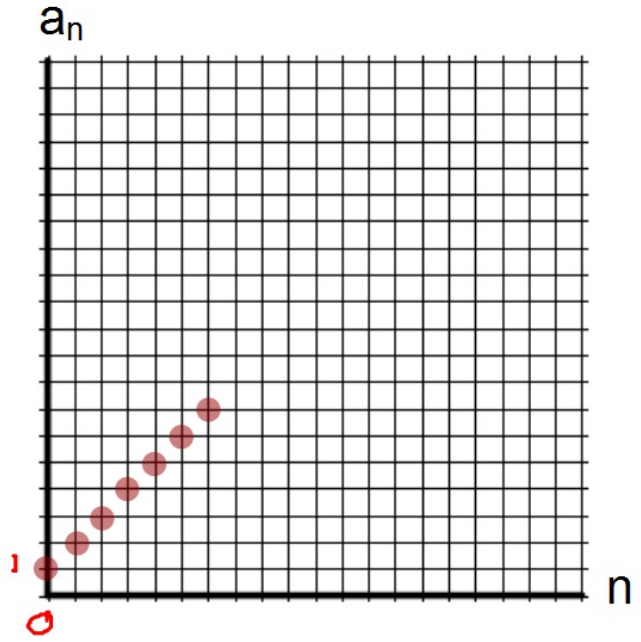
EF: $a_n = 4 + (n-1)(-2)$

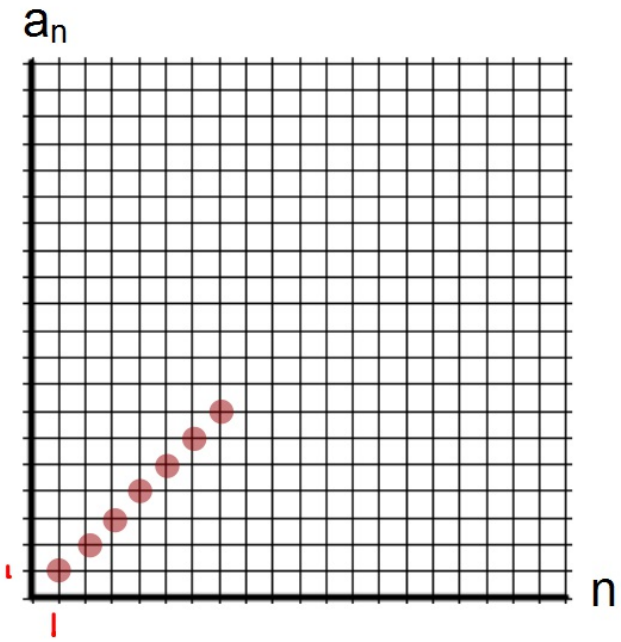
SF: $a_n = -2n + 6$ $\quad a_{50} = -94$

RF: $a_n = a_{n-1} - 2$

IWBAT use the simple, explicit, or recursive formula to solve problems about arithmetic sequences; find the missing term in an arithmetic sequence; find the common difference between terms of an arithmetic sequence; and identify the graph that represents a given arithmetic sequence.

Which of these is the graph of the sequence 1, 2, 3, 4, ...?





A sequence will always have a discrete graph.

IWBAT use the simple, explicit, or recursive formula to solve problems about arithmetic sequences; find the missing term in an arithmetic sequence; find the common difference between terms of an arithmetic sequence; and identify the graph that represents a given arithmetic sequence.

4.2 Arithmetic Sequences

5/13/14

Vocabulary 4.2.1 p. 24

Practice 4.2.2

Apex quizzes 4.2.3 & 4.2.4

IWBAT use the simple, explicit, or recursive formula to solve problems about arithmetic sequences; find the missing term in an arithmetic sequence; find the common difference between terms of an arithmetic sequence; and identify the graph that represents a given arithmetic sequence.

Find the missing term in an arithmetic sequence.

A) 25, 20, 15, 10, ... (−5, −5)

B) 3, 10, 17, 24, ... $\frac{24-3}{3}=7$

C) 5, 14, 23, 32, 41, ... $\frac{41-5}{4}=9$

4.3 Geometric Sequences

5/14/14

Define and identify a geometric sequence.

A geometric sequence is a sequence of numbers where you multiply your terms by a number to get the next term. This number is called the common ratio.

Which of the following are geometric sequences?

1) 1, 2, 4, 8, ... (×2, ×2) yes $r = 2$

2) 4, 2, 1, 0.5, ... ($\times\frac{1}{2}$, $\times\frac{1}{2}$) yes $r = \frac{1}{2}$

3) 2, -4, 8, ... (×−2, ×−2) yes $r = -2$

4) 1, 4, 7, ... (+3, +3) NO

How do you know?

IWBAT use the recursive formula to find a term in a geometric sequence, find the missing term in a geometric sequence, identify the explicit and recursive formulas for geometric sequences, find the common ratio between terms of a geometric sequence, analyze the graph of a geometric sequence. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

4.3 Geometric Sequences

5/14/14

Find the rule for the following sequence:

2, -4, 8, ... multiply by -2

$a_n = a_{n-1} \cdot -2$

$a_n = a_{n-1} \cdot r$

What is the 4th term? What is the 50th term?

$a_4 = a_3 \cdot -2$ $\qquad$ $a_{50} = a_{49} \cdot -2$

$a_4 = 8 \cdot -2 = -16$

IWBAT use the recursive formula to find a term in a geometric sequence, find the missing term in a geometric sequence, identify the explicit and recursive formulas for geometric sequences, find the common ratio between terms of a geometric sequence, analyze the graph of a geometric sequence.

Explicit formula: $a_n = a_1 * r^{n-1}$

Recursive formula: $a_n = r * a_{n-1}$

Write the explicit formula and the recursive formula for the following sequence: 4, 2, 1, 0.5, ...

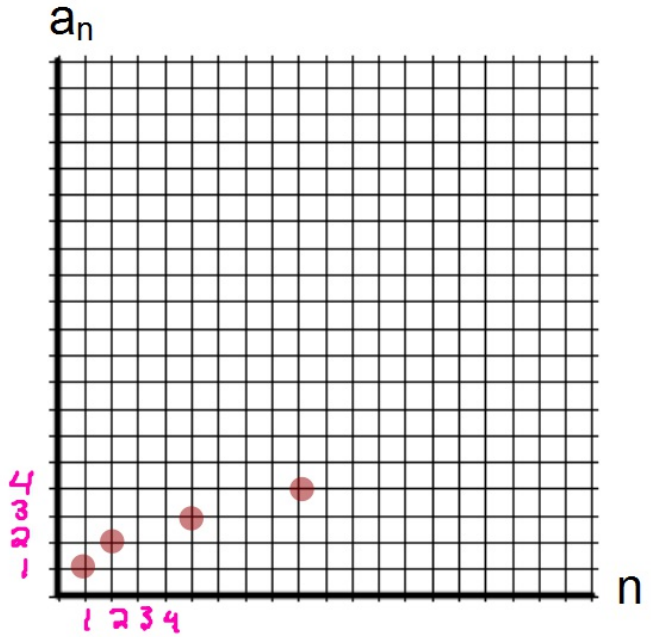
$r = \frac{1}{2}$

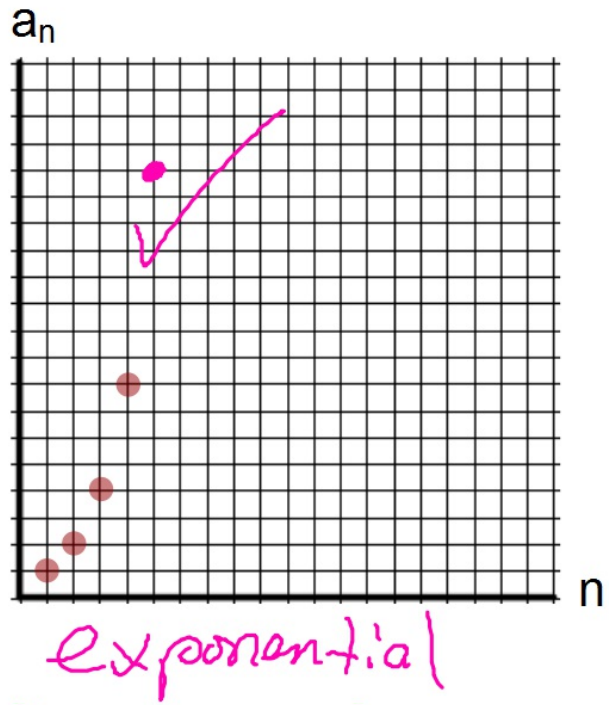
RF: $a_n = \frac{1}{2} a_{n-1}$

EF: $a_n = 4\left(\frac{1}{2}\right)^{n-1}$

IWBAT use the recursive formula to find a term in a geometric sequence, find the missing term in a geometric sequence, identify the explicit and recursive formulas for geometric sequences, find the common ratio between terms of a geometric sequence, analyze the graph of a geometric sequence.

Which of these is the graph of the sequence 1, 2, 4, 8, ...?





A sequence will always have a discrete graph.

$f(x) = a \cdot b^x$

$a_n = a_1 r^{n-1}$

Vocabulary 4.3.1 p. 22

Practice 4.3.2

Apex quizzes 4.3.3 & 4.3.4

IWBAT use the recursive formula to find a term in a geometric sequence, find the missing term in a geometric sequence, identify the explicit and recursive formulas for geometric sequences, find the common ratio between terms of a geometric sequence, analyze the graph of a geometric sequence.

Identify the geometric sequences and their common ratios.

A) 1, 4, 9, 16, ... no

B) 3.14, 6.28, 9.42, 12.56, ... no

C) 0.5, 0.1, 0.02, ... yes $r = \frac{1}{5}$

D) $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \cdots$ yes $r = \frac{2}{3}$

4.4 Applications of Number Sequences

5/15/14

Identify an arithmetic sequence that represents a real-life application.

Simple Interest

$I = Prt$ $\qquad$ $P_t = P + Prt$

\$400 $\quad$ 4.1% $\quad$ 10 yr

$P_{10} = 400 + 400(10)(0.041)$

$P_{10} = \$564$

4.4 Applications of Number Sequences 5/15/14

IWBAT identify the formula for the nth term of an arithmetic sequence, identify a geometric sequence that represents a real-life application, identify the formula for the nth term of a geometric sequence, and identify a Fibonacci sequence and a sequence that represents pyramidal numbers. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

Recall:

Arithmetic

Explicit formula: $a_n = a_1 + (n-1) * d$

Simple formula: $a_n = d * n + (a_1 - d)$

Recursive formula: $a_n = a_{n-1} + d$

Geometric

Explicit formula: $a_n = a_1 * r^{n-1}$

Recursive formula: $a_n = r * a_{n-1}$

IWBAT identify the formula for the nth term of an arithmetic sequence, identify a geometric sequence that represents a real-life application, identify the formula for the nth term of a geometric sequence, and identify a Fibonacci sequence and a sequence that represents pyramidal numbers.

4.4 Applications of Number Sequences 5/15/14

Explicit formula for a constant velocity situation

$$x_n = x_1 + (n-1)(v\Delta t)$$

You are playing paintball with a friend who is standing 25 m away from you. You shoot at her and miss. The paintball has a horizontal velocity of 150 m per second. Which of the following sequences describes the distances of the ball from you (in meters) at intervals of one-tenth of a second, starting when it swooshes by her?

- A. 15, 30, 45, 60, 75, ...
- B. 25, 175, 325, 475, 625, ...
- C. 25, 40, 55, 70, 85, ...
- D. 150, 300, 450, 600, 750, ...
- E. 15, 25, 35, 45, 55, ...

$X_1 = 25m$

$V = 150\ m/s$

$\Delta t = \frac{1}{10}\ s$

$X_n = 25 + (n-1)(150 \cdot \frac{1}{10})$

$X_n = 25 + 15(n-1)$

IWBAT identify the formula for the nth term of an arithmetic sequence, identify a geometric sequence that represents a real-life application, identify the formula for the nth term of a geometric sequence, and identify a Fibonacci sequence and a sequence that represents pyramidal numbers.

4.4 Applications of Number Sequences 5/15/14

Explicit formula for compound interest

$$P_n = P_1(1+i)^{n-1}$$ Do Not Use on Test

If you invest $400 in a savings account with an annual interest rate of 4.1% for 10 years, what will be the final balance?

$P_{10} = 400(1+.041)^{10}$

$P_{10} = \$597.81$

$$P_t = P_0\left(1+\frac{r}{n}\right)^{nt} \qquad P_t = Pe^{rt}$$

IWBAT identify the formula for the nth term of an arithmetic sequence, identify a geometric sequence that represents a real-life application, identify the formula for the nth term of a geometric sequence, and identify a Fibonacci sequence and a sequence that represents pyramidal numbers.

4.4 Applications of Number Sequences 5/15/14

A particular strain of bacteria doubles in population every 10 minutes. Assuming you start with a single bacterium in a petri dish, how many bacteria will there be in 2.5 hours?

$B_0 = 1 \quad r = 2 \quad t = 2.5h \quad i = 10min$

$B = 1 \cdot 2^{15} = 32,768$

A certain bacterial culture is multiplying in a culture dish, doubling in size every 5 minutes. It takes exactly an hour to fill the whole dish. How long does it take to fill half the dish?

$d = 5min$

$60min - 5mn = 55min$

IWBAT identify the formula for the nth term of an arithmetic sequence, identify a geometric sequence that represents a real-life application, identify the formula for the nth term of a geometric sequence, and identify a Fibonacci sequence and a sequence that represents pyramidal numbers.

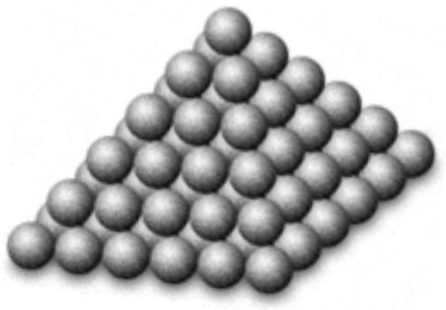
4.4 Applications of Number Sequences

5/15/14

Pyramidal Numbers Sequence

1, 5, 14, 30, 55, ...

$+2^2 + 3^2 + 4^2 + 5^2 + \ldots$



$$Pyr_9 = \frac{9(10)(19)}{6} = 285$$

Square Pyramid

Explicit formula

$$\text{Pyr}_n = \frac{n(n+1)(2n+1)}{6}$$

IWBAT identify the formula for the nth term of an arithmetic sequence, identify a geometric sequence that represents a real-life application, identify the formula for the nth term of a geometric sequence, and identify a Fibonacci sequence and a sequence that represents pyramidal numbers.

4.4 Applications of Number Sequences

5/15/14

Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

$a_n = a_{n-1} + a_{n-2}$ 1414

Real-life examples: 4.4.1 p. 26 & 28

IWBAT identify the formula for the nth term of an arithmetic sequence, identify a geometric sequence that represents a real-life application, identify the formula for the nth term of a geometric sequence, and identify a Fibonacci sequence and a sequence that represents pyramidal numbers.

Vocabulary 4.4.1 p. 32

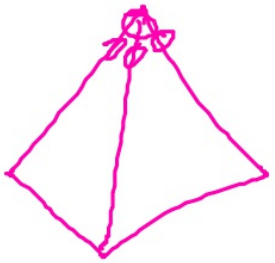
Practice 4.4.2

Apex quizzes 4.4.3, 4.4.4, & 4.4.5

IWBAT identify the formula for the nth term of an arithmetic sequence, identify a geometric sequence that represents a real-life application, identify the formula for the nth term of a geometric sequence, and identify a Fibonacci sequence and a sequence that represents pyramidal numbers.

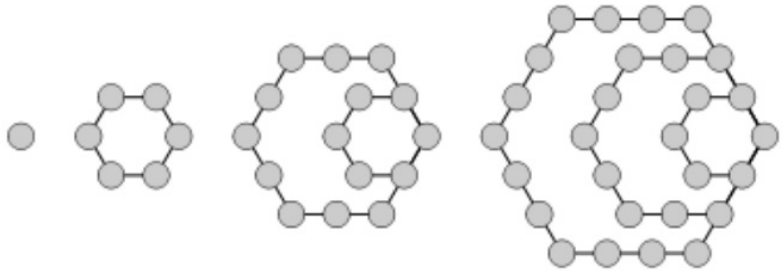
Identify a sequence that represents pyramidal numbers.

A) 1, 4, 9, 16, 25, ...



B) 1, 5, 14, 30, 55, ... Square Pyramid

C) 1, 7, 22, 50, 95, ... Hexagonal Pyramid



1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276, 325, 378, 435, 496, 561, 630, 703, 780, 861,

4.5 Number Series

5/16/14

Define finite series, infinite series, convergent series, and divergent series.

Infinite series - does not end

finite - has an end

Convergent - numbers are coming together, getting closer to zero

divergent - numbers get further from zero

IWBAT convert between the summation notation of a series and its expanded form; expand and evaluate the summation notation of a series for a given value of n; find the sum of a finite arithmetic series; find the sum of an infinite geometric series; and find the first and last terms, the common ratio, or the number of terms of a given geometric series. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.

4.5 Number Series

5/16/14

Summation notation $\sum_{i=1}^{n} f(x)$

Expand

$$\sum_{i=1}^{n} f(x) = f(1) + f(2) + \ldots + f(n)$$

Expand and evaluate

$$\sum_{i=1}^{5} 2i = 2(1)+2(2)+2(3)+2(4)+2(5)$$

$$2+4+6+8+10 = 30$$

IWBAT convert between the summation notation of a series and its expanded form; expand and evaluate the summation notation of a series for a given value of n; find the sum of a finite arithmetic series; find the sum of an infinite geometric series; and find the first and last terms, the common ratio, or the number of terms of a given geometric series.

4.5 Number Series

5/16/14

Find the first and last terms, the common ratio, and the number of terms of a given geometric series.

$$\sum_{i=1}^{5} (2)^{i+1}$$

$n = 5$

$2^{1+1} = 2^2 = 4$

$2^{5+1} = 2^6 = 64$

$r = 2$

$$\sum_{i=1}^{6} (-2)^{i-1}$$

$r = -2$

$n = 6$

$-2^{1-1} = -2^0 = 1$

$-2^{6-1} = -2^5 = -32$

IWBAT convert between the summation notation of a series and its expanded form; expand and evaluate the summation notation of a series for a given value of n; find the sum of a finite arithmetic series; find the sum of an infinite geometric series; and find the first and last terms, the common ratio, or the number of terms of a given geometric series.

4.5 Number Series

5/16/14

Find the sum of a finite arithmetic series.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$n = \frac{a_n - a_1}{d} + 1$$

A) 1 +2 + 3 +... +12

$n = \frac{12-1}{1} + 1 = 12$

$S_n = \frac{12}{2}(1+12) = 6(13) = 78$

B) 5 +8 +11 +... +50

$n = \frac{50-5}{3} + 1 = \frac{45}{3} + 1 = 15 + 1 = 16$

$S_n = \frac{16}{2}(50+5) = 8(55) = 440$

IWBAT convert between the summation notation of a series and its expanded form; expand and evaluate the summation notation of a series for a given value of n; find the sum of a finite arithmetic series; find the sum of an infinite geometric series; and find the first and last terms, the common ratio, or the number of terms of a given geometric series.

Find the sum of a finite geometric series.

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$1 + \left(\frac{\log\left(\frac{a_n}{a_1}\right)}{\log(r)} \right) = n$$

A) 2 +6 +18 +... +13,122 $a_n = 2 \cdot 3^n$

$S_n = \frac{2(1-3^9)}{(1-3)}$

$S_n = 19,682$

$\log\left(\frac{13122}{2}\right) = (3^n)\log$

$\frac{\log(6561)}{\log 3} = n \frac{\log 3}{\log 3}$

B) 2 +1 +.5 +.25 +... +0.015625

$S_n = \frac{2\left(1-\frac{1}{2}^8\right)}{\left(1-\frac{1}{2}\right)} = 3.984$

$n = 8 + 1 = 9$

IWBAT convert between the summation notation of a series and its expanded form; expand and evaluate the summation notation of a series for a given value of n; find the sum of a finite arithmetic series; find the sum of an infinite geometric series; and find the first and last terms, the common ratio, or the number of terms of a given geometric series.

The sum of an infinite arithmetic series cannot be calculated as it is always ∞ or $-\infty$.

The sum of a divergent infinite geometric series cannot be calculated as it is always ∞ or $-\infty$.

Find the sum of a convergent infinite geometric series. $S_\infty = \frac{a_1}{1-r}$

$$2\frac{1}{4} + 1\frac{1}{2} + 1 + \frac{2}{3} + \ldots$$

$$S_\infty = \frac{\frac{9}{4}}{1-\frac{3}{3}} = \frac{\frac{9}{4}}{\frac{1}{3}} = \frac{9}{4}\cdot\frac{3}{1} = \frac{27}{4} = 6\frac{3}{4}$$

IWBAT convert between the summation notation of a series and its expanded form; expand and evaluate the summation notation of a series for a given value of n; find the sum of a finite arithmetic series; find the sum of an infinite geometric series; and find the first and last terms, the common ratio, or the number of terms of a given geometric series.

Vocabulary 4.5.1 p. 28

Practice 4.5.2

Apex quizzes 4.5.3 & 4.5.4

IWBAT convert between the summation notation of a series and its expanded form; expand and evaluate the summation notation of a series for a given value of n; find the sum of a finite arithmetic series; find the sum of an infinite geometric series; and find the first and last terms, the common ratio, or the number of terms of a given geometric series.

4.6 Sequences & Series Wrap-up

5/19/14

Find the sum of a finite arithmetic series.

A) $3 + 5 + 7 + \ldots + 227$

$n = \frac{227-3}{2} + 1 = 113$

$S_n = \frac{113}{2}(227+3) = 12{,}995$

B) $\sum_{i=1}^{19} 7i + 5$ $\quad 19 = n$

$n = 19$

$S_n = \frac{19}{2}(12 + 138)$

$S_n = 1{,}425$

$S_n = \frac{n}{2}(a_1 + a_n)$

Complete unit test 4.6.2.

IWBAT assess my learning in this unit. I will capture my thinking using the math note catcher including teacher and student-team modeled example problems on the Promethean board. I will demonstrate my understanding on my exit ticket.